**Numerically-analysed Multiwavelet Transform computations: multidimensional compression case studies**

Nada Alramahi,

*School of Engineering,*

*London South Bank University*

*nadaalramahi@yahoo.com*

Martin Bush,

*School of Engineering,*

*London South Bank University*

*martin.bush@lsbu.ac.uk*

M. Rafiq Swash,

*Dept of Electronic and Computer Engineering,*

*Brunel University London*

*rafiq.swash@brunel.ac.uk*

**Abstract**

*Multiwavelet is a signal processing transform that may be utilised in image compression. The main aim of this paper is investigated a practical approach of studying the 3D stereoscopic film compression. As to our best of knowledge, a straightforward mechanism exists, which autonomously deal with the frames context and analysis it’s left and right components via compressing to publish and display a set of connected authorised frames. However, various aspects and challenging issues that require extensive research to cope with and to be considered during the research to provide a real time and effective approach (Al-Ramahi, Al-Bayatti, & Alfaouri, 2007) [2].*

*The coding of Multiwavelet Transform (MWT) is based on the fact that the huge sequences of images in the 3D film are contiguous on the temporal axis. Therefore, the MWT may completely exploit the correlations of images by processing with orthogonal filters and the set partitioning techniques. That includes a dynamic coding of the coefficients to implement the Rate-distortion performance indices of Peak Signal to Noise Ratio (PSNR) and Mean Square Error (MSE). Additionally, the colour space relationships are exploited as well as maintaining the full embeddedness required by colour image sequences compression ratio [3]. MWT is utilised to address the issue of compression and manage a precise action, encourage, enhance the film image by improving the signal of the images film.*

*Multiwavelet Transform produces a new image with a minimum size of the original image to avoid the overhead and create a new film with minimum size [1] [2]. The performance analysis depends on watching the parsing assessment of compression. What's more, the significance of comment to improve the execution of the 3D film quality is evaluated too [3] [4]. This method efficiently outperforms the existing work of compressing 3D stereoscopic films [5] [6]. All the objectives of this project are carried out and the results clearly obtained by simulation without losing the data and enhance the quality.*

***Keywords****: Stereoscopic Film; Film Compression; Multiwavelet (MWT); Inverse Multiwavelet (IMWT); broadcasting time; Compressibility ratio.*

1. **Introduction**

Multiwavelets are valuable devices for signal handling applications, for example, image compression and denoising [1] [2] [7]. Up to this point, just scalar wavelets were known, these are wavelets produced by one scaling function. Be that as it may, one can envision a circumstance when there is more than one scaling function [2] [3] [8]. This prompts to the development of multiwavelets, which have a few favourable circumstances in contrast with scalar wavelets [9]. Such elements as short support, orthogonality, symmetry, and vanishing minutes are known to be vital in signal preparing. However, a scalar wavelet can have only part of these properties [9]. Then again, a multiwavelet framework can all the while give consummate remaking while safeguarding length (orthogonality), great execution at limits (through direct stage symmetry), and a high request of guess (vanishing minutes). Therefore, multiwavelets offer the likelihood of unrivalled execution for image preparing applications contrasted scalar wavelets [1] [2] [5] [10].

Multiwavelet work on multi-level decompositions is similarly performed. The multiwavelet decompositions iterate on the low-pass coefficients from the previous decomposition. In scalar wavelets case, the low-pass quarter image is a single sub-band. But when the multiwavelet transform is used, the quarter image of low-pass coefficients is actually a 2×2 block of Li Lj sub-bands. Owing to the environment of the preprocessing and symmetric extension method, data in these different sub-bands becomes intermixed during the iteration of the multiwavelet transform [11]. The intermixing of the multiwavelet low-pass sub-bands leads to suboptimal results [8].

Consider the multiwavelets transform coefficients resulting from a single-level decomposition. It can be readily observed that the 2×2 “low-pass” block (upper left corner) actually contains one low-pass sub-band and three bandpass sub-bands. The L1L1 sub-band is a smaller version of the original image.

1. **Multiwavelet Transform**

One of the imperative contrasts amongst multiwavelets and scalar wavelets is that every channel in the filter bank has a vector-esteemed information and a vector-esteemed yield. A scalar-esteemed information signal should some way or another be changed over into a reasonable vector-esteemed flag. This transformation is called pre-processing [1] [2].

A few purposes behind possibly picking multiwavelets can be abridged as takes after [8] [9]:

1. The additional degrees of flexibility intrinsic in multiwavelets can be utilised to lessen limitations on the filter properties. For instance, it is notable that a scalar wavelet cannot at the same time have both orthogonality and a symmetric drive reaction that has a length more noteworthy than 2. Symmetric filters are vital for symmetric signal expansion, while orthogonality makes the change less demanding to outline and actualize.
2. The bolster length and the quantities of vanishing minutes are straightforwardly connected to the filter length for scalar wavelets. This implies longer filter lengths are required to accomplish higher request of estimation to the detriment of expanding the wavelet's interim of support. A high request of an estimate is craved for better coding pick up, however, shorter wavelet support is by and large liked to accomplish a superior confined guess of the info work.
3. As opposed to the restrictions of scalar wavelets, multiwavelets can have the best of every one of these properties all the while. For instance, the GHM multiwavelet [1] [2] [8] is orthogonal, has the second request of an estimate, has symmetric scaling and wavelet capacities (and in this way symmetric filters), and has short support for both of its scaling capacities. This mix of good properties is outlandish with scalar wavelets.
4. One attractive component of any transform as a part of image compression is the measure of vitality compaction accomplished. A filter with great vitality compaction properties can de-correlate a genuinely uniform info motion into a little number of scaling coefficients containing the vast majority of the vitality and a substantial number of inadequate wavelet coefficients. This gets to be distinctly critical amid quantization since the wavelet coefficients are normally spoken to with fundamentally fewer bits all things considered than the scaling coefficients [10] [12]. In this way, better execution is acquired when the wavelet coefficients have values bunched around zero with little change, to maintain a strategic distance from however much quantization noise as could be expected. Some multiwavelets accomplish fundamentally preferable vitality compaction over some scalar wavelets. In this manner, multiwavelets can offer better revamping quality at a comparative piece rate.
5. Previous writing has demonstrated promising outcomes in the use of multiwavelets to image compression. No less than one multiwavelet drastically beat scalar wavelets on an engineered test image.
6. Finally, there is the subject of computational multifaceted nature. At first look doubtlessly scalar wavelets have the reasonable preferred standpoint since every branch in a multiwavelet filter bank has two channels and 2-input, 2-yield channels [8]. In any case, each of the filters in a symmetric-antisymmetric multifilter has a similar sort of symmetry that makes the symmetric biorthogonal multiwavelets proficient [1] [2] [8]. Additionally, every filter in a multi-filter framework forms less information than a filter in a scalar filter bank at a similar level (due to the pre-handling talked about later). With a similar number of disintegration levels, it is clear that the multiwavelets require generally twice as much calculation. Be that as it may, multiwavelets still think about positively on the grounds that they can give execution tantamount to scalar wavelets with shorter filters of the length of a half. Along these lines for a similar nature of deteriorated levels, multiwavelets require about a large portion of the quantities of operations [9] [10] [12].
7. **Geronimo, Hadrian, and Massopust (GMH) Multiwavelet Filter**

Wavelet has an associated scaling function $φ(t)$ and wavelet function ψ(t), multiwavelets have two or more scaling and wavelet functions. Where, the set of scaling functions may be written using the vector notation $φ\left(t\right)=[φ\_{1}\left(t\right), φ\_{2}\left(t\right)… φ\_{r}(t)]^{T}$, where $φ(t)$ is called the multiscaling function. Similarly, the multiwavelet function is defined from the set of wavelet functions as $Ψ\left(t\right)=[Ψ\_{1}\left(t\right), Ψ\_{2}\left(t\right)… Ψ\_{r}(t)]^{T}$. $Ψ(t)$ is called wavelet or scalar wavelet when r=1. While in principle r can be arbitrarily large. The multiwavelets surveyed to date are primarily for r=2 [1] [2] [8].

The multiwavelet two-scale equations resemble those for scalar wavelets:

$$φ\left(t\right)=\sqrt{2}\sum\_{k=-\infty }^{\infty }H\_{k}φ\left(2t-k\right) …1$$

$$Ψ\left(t\right)=\sqrt{2}\sum\_{k=-\infty }^{\infty }G\_{k}φ\left(2t-k\right) …2$$

Where Hk and Gk are (r × r) matrix filters for each integer k. The matrix elements in these filters give a larger number of degrees of freedom than a standard scalar wavelet. These additional degrees of freedom can be utilised to join valuable properties into the multiwavelet filters, for example, orthogonality, symmetry, and high order of estimation. The key, then, is to make sense of how to make the best utilization of these additional degrees of freedom. Multifilter development strategies are as of now being produced to adventure them. Be that as it may, the multi-channel nature of multiwavelets likewise implies that the sub-band structure coming about because of passing a signal through a multifilter bank is distinctive. Adequately unique, built up quantization techniques may be executed efficiently with wavelets, however, not likely with multiwavelets [8] [9].



One famous multiwavelet filter is the GHM filter proposed by Geronimo, Hardian, and Massopust [1] [2] [8]. The GHM basis offers a mixture of orthogonality, symmetry, and compact support, which cannot be accomplished by any scalar wavelet basis [8]. According to Eqs. (1) and (2) the GHM two scaling and wavelet functions satisfy the following two-scale dilation equations:

$$\left[\begin{matrix}φ\_{1 }\left(t\right)\\φ\_{2}\left(t\right)\end{matrix}\right]= \sqrt{2}\sum\_{k}^{}H\_{k}\left[\begin{matrix}φ\_{1 }\left(2t-k\right)\\φ\_{2}\left(2t-k\right)\end{matrix}\right] …3$$

$$\left[\begin{matrix}φ\_{1 }(t)\\φ\_{2}(t)\end{matrix}\right]= \sqrt{2}\sum\_{k}^{}G\_{k}\left[\begin{matrix}φ\_{1 }(2t-k)\\φ\_{2}(2t-k)\end{matrix}\right] …4$$

Where Hk are four scaling matrices as H0, H1, H2, and H3, [8],

, ,

… (5)

, ,

Also, four wavelet matrices high pass filter G0, G1, G2, and G3 [8]:

, ,

… (6)

, 

Using a similar procedure done for the scalar wavelet by using iteration scheme delineated by Eqs. (1) and (2), to draw the scaling and wavelet function for the GHM multiwavelets [8]. Thus, there are two scaling functions and two wavelets functions beginning from two box functions as shown in Fig.1.

1. **Multiwavelet Filter Analysis and Synthesis**

The multiwavelet transformations are directly applicable only to one-dimensional (1-D) signals. But images are two-dimensional (2-D) signals, so there must be a way to process them with a 1-D transform [8] [9]. The two main methods for doing this are separable and non-separable algorithms. Separable methods simply work on each dimension in series. The typical approach is to process each of the rows in order and then process each column of the result which it is easier to see the results of the computing steps so it has adopted in this project, like an Over-Sampled Scheme of Preprocessing (Repeated Row Preprocessing) [1] [2] [9]. Non-separable methods work in both image dimensions at the same time. While non-separable methods can offer saving in computation, but, more difficult to implement, e.g., a critically-sampled scheme of preprocessing [8] [9].

Where Hi and Gi are the low- and high-pass filter impulse responses. They are 2x2 matrices which can be written as follows:



… (7)

By examining the transform matrices of the scalar multiwavelets as shown in Eqs. (6) respectively, one can see that in the multiwavelets transform domain there are first and second low-pass coefficients followed by first and second high pass filter coefficients rather than one low-pass coefficient followed by one high-pass coefficient. Hence, there are four sub-bands in the transform domain for each separate four coefficients are shown in Fig. 2. [1] [2] [8].

 

**Fig. 2: Multiwavelet Sub-bands after a Single-LevelDecomposition**

That is mean multiwavelet setting, GHM multiscaling and multiwavelets functions coefficients are 2×2 matrices, and during transformation step they must multiply vectors (instead of scalars). This means that multifilter bank need 2 input rows. So the most obvious way to get two input rows from a given signal is to repeat the signal [8]. Two rows go into the multifilter bank. This procedure is called “Repeated Row” which introduces oversampling of the data by a factor of 2 [1] [2] [8].

For a given scalar input signal {Xk} of length N, a row preprocessing of this signal is by repeating the input stream with the same stream multiplied by a constant α. So the input length of two vector are formed from the original as [8] [10] [12],

… (8)

$\left[ \begin{matrix}x\_{k}\\αx\_{k}\end{matrix} \right]$ where k=0,1, 2,…,N-1

Where Xk = C = constant, for all k, then the output from the high-pass multifilter is zero. This can always be done if the system has approximation order higher than zero. For the GHM case, α=1/√2 is selected since [1] [2] [8]:

 $\left[G\_{1}+G\_{2}+G\_{3}+G\_{4}\right]\left[\begin{matrix}C\\\frac{C}{√2}\end{matrix}\right]=\frac{1}{10}\left[\begin{matrix}8&-\frac{10}{√2}\\0&0\end{matrix}\right] \left[\begin{matrix}C\\ \frac{C}{\sqrt{2}}\end{matrix} \right]=\left[\begin{matrix}0 \\0\end{matrix}\right]$

… (9)

… (10)

$$\left[H\_{1}+H\_{2}+H+H\_{4}\right]\left[\begin{matrix}C\\\frac{C}{\sqrt{2}}\end{matrix}\right]=\frac{1}{5 }\left[\begin{matrix}\frac{6}{√2}&4\\4&√2\end{matrix}\right] \left[\begin{matrix}C\\ \frac{C}{\sqrt{2}}\end{matrix} \right]=\sqrt{2}\left[\begin{matrix}C\\ \frac{C}{\sqrt{2}}\end{matrix}\right]$$

After the multiwavelet reconstruction (synthesis) step a post filtering is applied.

Previously mentioned, a generic procedure can be developed for computing a single-level 2-D DMWT using GHM four multi-filters and repeated row pre-processing.

By utilising an over-sampled outline of repeated row pre-processing, the DMWT matrix is doubled in dimension compared with that of a square matrix N×N input. Transformation matrix dimensions equal image dimensions after pre-processing.

A couple of conclusions may be drawn from these observations:

* Since these four LL sub-bands possess different statistical characteristics, mixing them together using the multiwavelet decomposition described previously results in further sub-bands with mixed data characteristics. This infers that typical quantization outlines either low-pass or high-pass will not give the best possible results.
* Since only the L1L1 sub-band actually has low-pass characteristics, there is only a need to perform further iterations on that one sub-band [1] [2] [8]. Thus iterating only on the L1L1 sub-band at each stage in the decomposition does yield better performance than iterating on the entire LL sub-band. It is also worth noting that iterating only on the L1L1 sub-band requires one-quarter of the computational complexity as iteration over the entire LL sub-band, thus improving run-time performance as well.

Fig.3 shows a 1-level sub-band decomposition and reconstruction framework for DMWT. Where the first part represents a 1-level multiwavelet decomposition, where a vector input stream is decomposed by a matrix low pass filter H, and a matrix high pass filter G, to generate the next lower resolution. Sub-sampling follows this by a factor of two to preserve the compact representation of the input signal. For octave bandwidth decomposition, only the low pass sub lower resolution. The second part shows the corresponding 1-level multiwavelet reconstruction. The sub-bands are first up-sampled by a factor of two before they are filtered by the synthesis matrix to recover the original vector stream.

1. **Examples to Computing 2D Matrix in MWT**

To verify the general procedure for computing single-level DMWT using repeated row preprocessing, let’s take a general 2-D signal, should be of length NxN, where N must be a power of two, for example, any 8×8 matrix, and apply the following steps [8]:

1. Let X be the input 2-D signal.

2. For an 8×8 matrix input 2-D signal, X, construct a transformation matrix W, using GHM low- and high-pass filters,

3. Apply row pre-processing to the input 2-D matrix, X, using repeated row pre-processing and α = 1/√2, …(11)

4. Apply row transformation

i. Let, [ z ]= [W]×[x] … (12)

ii. Permute [z] … (13)

5. Apply column transformation,

i. Transpose [p] matrix. … (14)

ii. Pre-process [p]t to get [P] matrix … (15)

iii. Let[b]=[W]×[P] … (16)

 iv. Permute [b] to get [B] matrix which is 16×16 matrix. … (17)

6. The final DMWT matrix [Y]results from the following steps,

i. Transpose [B] matrix to get [y] matrix … (18)

ii. The four basic sub-bands of a matrix [y] may be obtained by coefficients permutation each… (19)

iii. Apply coefficients permutation to each sub band of [y] alone to get 16 sub-band in [Y] matrix as follows: … (20)

So, [Y] is 16 sub band the final single-level MWT matrix.

1. **Inverse Discrete Multiwavelet Transform Algorithms**

To reconstruct the original 2-D signal (N×N matrix) from the discrete multiwavelets transformed 2-D signal (2N×2N matrix for an over-sampled scheme of preprocessing or Repeated Row Post-processing), the Inverse Discrete Multiwavelets Transform (IDMWT) should be used. Next section will explain the step [8]**:**

1. **Example for Computing Inverse DMWT**

The following example shows how the inverse transform process works on a N×N matrix.

To verify IDMWT procedure, apply it to the 8×8 matrix, [Y], given in (20) as the 2N×2N over-sampled pre-processed DMWT matrix to reconstruct the 4×4 matrix, [X], given in red matrix of 3D as the N×N original 2-D signal matrix as follows [8]:

1. Apply coefficients shuffling to each sub-band of [Y] matrix (20) which results in [y] matrix (20) by the steps of coefficients shuffling process described above.

2. Column reconstruction applied now on [y] (20) matrix,

i. Transpose [y] to get [B] matrix given in (19).

ii. Apply shuffling to [B] matrix given in (19) to have [b] matrix given in (19) as a result of shuffling.

iii. Using [W] matrix given in (7),

… (21)

 [P] = [W]t ×[b]

[P] is given in (15).

1. Post processing [P] matrix results in [p]t given in (15).

4. Row reconstruction applied on [p]t matrix,

i. Transpose [p]t matrix (14) which results in [p] matrix (13).

ii. Apply shuffling to [p] matrix given in (13) to have [z] matrix given in (13) as a result of shuffling.

iii. Using [W] matrix given in (7),

 [x] = [W]t ×[z]

… (22)

[x] is given in (12).

5. Post-processing [x] matrix results in [X] given in (11) which is the original reconstructed 2-D signal.

1. **Conclusion**

 This paper focuses on the benefit of Multiwavelet Transform (MWT) to create a new small version or compress part of a 3D film. The inverse- Multiwavelet Transform (IMWT) is used in the decompression part. Theoretical and experimental results have been steadily obtained.

 The approximation order and regularity are very important for some applications such as digital signal processing applications, but in 3D film compression, the effect of approximation order and regularity is still unknown. Generally, after the presentation of a pre-filtering technique, multiwavelets with multiplicity 2 may be successfully applied in compression application.

 Balanced multiwavelets, unlike the scalar wavelets, combine the two desirable properties of orthogonality and symmetry. Recently developed balanced multiwavelets have also proven to be effective for compression. However, their performance falls slightly short of the scalar wavelets. Another property of multiwavelets: the perfect reconstruction property which shows mainly the performance differences between the scalar wavelets and balanced multiwavelets.

1. **References**

|  |  |
| --- | --- |
| [1]  | N. Al-Ramahi, M. Alfaouri and . H. Al-Bayatti, "Novel Techniques for Face Recognition Identification and Labeling," *International Journal of Soft Computing,* pp. 2(1), pp.129-137., 2007.  |
| [2]  | N. N. Al-Ramahi and M. Alfaouri, "New Techniques for Face Image Recognition Identification and Labeling," *i-Manager's Journal on Software Engineering,* vol. 1, no. 4, p. 62, 2007.  |
| [3]  | B. Alagendran and S. Manimurugan, "A survey on various medical image compression techniques," *International Journal of Soft Computing and Engineering (IJSCE) ISSN,* vol. 2, no. 1, pp. 2231-2307, 2012.  |
| [4]  | M. C. Forman, Compression of integral three-dimensional television pictures (Doctoral dissertation, De Montfort University), 1999.  |
| [5]  | B. Kaufmann and M. Akil, "3D images compression for multi-view auto-stereoscopic displays," in *In Computer Graphics, Imaging and Visualisation*, 2006 International Conference on IEEE, 2006.  |
| [6]  | T. Kesavamurthy and S. Rani, "Dicom Color Medical Image Compression using 3D-SPIHT for Pacs Application," *International journal of biomedical science,* vol. 4, no. 2, p. 113, 2008.  |
| [7]  | K. Köse, 3D model compression using image compression based methods, Doctoral dissertation, bilkent university, 2007.  |
| [8]  | H. N. Al-Taai, Computationally Efficient Wavelet Based Algorithms for Optical Flow Estimation, (Doctoral dissertation), Ph. D. Thesis, Univ. of Technology, Electrical and electronic engineering, Dep., 2005.  |
| [9]  | V. Strela and A. T. Walden, Signal and image denoising via wavelet thresholding: orthogonal and biorthogonal, scalar and multiple wavelet transforms, Nonlinear and Nonstationary Signal Processing, 1998.  |
| [10]  | N. N. Al-Ramahi, W. A. Al-Jowher and M. Alfaouri, "Image identification and labelling using hybrid transformation and neural network," *Neural Network World,* vol. 17, no. 4, p. 377, 2007.  |
| [11]  | M. Alfaouri, "Image Recognition Using Combination of Discrete Multi\_Wavelet and Wavenet Transform," *American Journal of Applied Sciences,* vol. 5, no. 4, pp. 418-426, 2008.  |
| [12]  | N. N. Al-Ramahi, M. Alfaouri and W. A. Al-Jouher,, "Automatic Image Identification and Labelling Using Wavelet Transform," *I-Manager’s Journal on Software Engineering,* vol. 1, no. 3, p. 24, 2007.  |
| [13]  | N. N. Al-Ramahi and H. M. Al-Bayatti, "Novel Techniques for Face Recognition Identification and Labeling," *Medwell Journals (International Journal of Soft Computing ISSN: 1816-9503),* pp. 216-224.  |